Forces on Submerged Objects and Buoyancy

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Mechanical Engineering 390

Fluid Mechanics

February 5, 2008

California State University Northridge

Outline

- Review last class
- Pressure on a vertical surface
- Pressure on a slanted surface
  - Average force due to pressure
  - Center of pressure
- Problem solving with forces
- Analysis and problem solving with buoyancy

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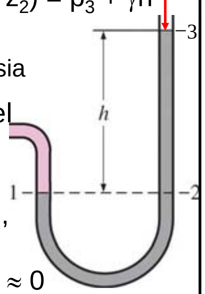
Review Manometer Problems

- Basic equation: pressures at two depths in same fluid: $p_2 = p_3 + \gamma(z_3 - z_2) = p_3 + \gamma h$
- "Open" means $p = p_{atm}$
  - p_{atm} = 101.325 kPa = 14.696 psia
- Same pressures at same level on two sides of a manometer
  - $p_2 = p_3$
- Watch units: in or ft, m or mm, psi or psf, N or kN
- For gases $\gamma \Delta z \approx 0$



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Review Basic Equations

- $dp/dz = -\gamma$

$$\int_{p_1}^{p_2} dp = p_2 - p_1 = -\int_{z_1}^{z_2} \gamma dz$$

- For incompressible fluid (constant γ)

$$p_2 + \gamma z_2 = p_1 + \gamma z_1$$

- For small elevation changes, Δz, we can assume that the density of a gas is constant

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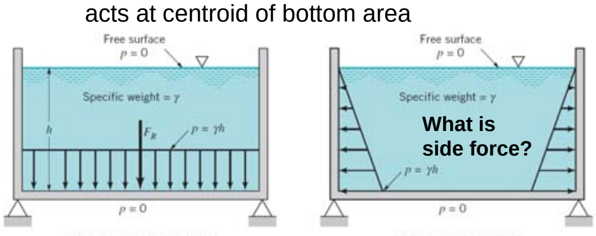
Pressure in a Vertical Tank

- Pressure distribution for flat surfaces
  - Resultant force at bottom, $F_R = pA = \gamma hA$, acts at centroid of bottom area



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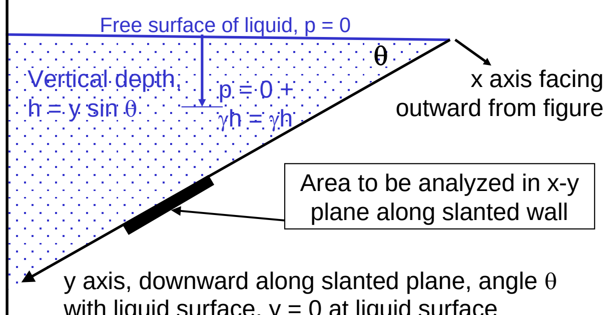
5

Slanted Surface



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Slanted Surface

$$h = y \sin\theta$$

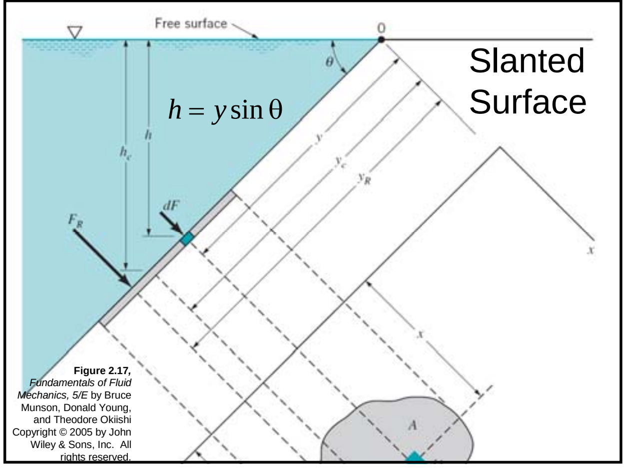


Figure 2.17, *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi

Slanted Surface

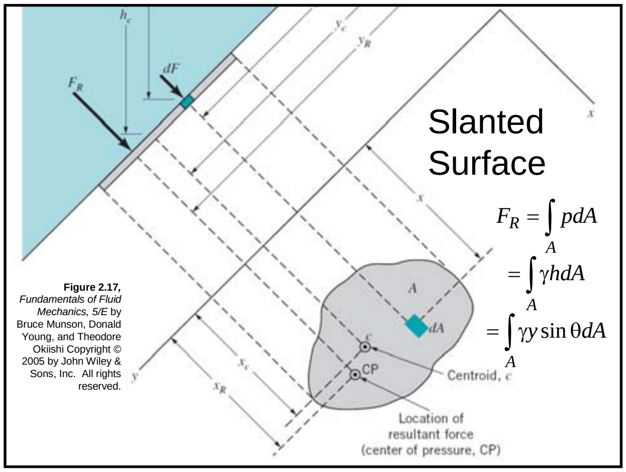


Figure 2.17, *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi

$$F_R = \int_A p\,dA = \int_A \gamma h\,dA = \int_A \gamma y \sin\theta\,dA$$

Resultant Force

$$F_R = \int_A \gamma y \sin\theta\,dA = \gamma \sin\theta \int_A y\,dA = \gamma y_c A \sin\theta$$

- Centroid, $y_c A = \int y\,dA$, used to compute F_R
- Center of pressure, **not y_c**, is ***location*** of resultant force
- Moment balance: $F_R y_{CP} = \int_A y\,dF$
  - Single force at point y_{CP} has same moment as integrated force over plate

Location of Resultant Force

- y_{CP} definition: $F_R y_{CP} = \int_A y\,dF$

$$F_R y_{CP} = \int_A y\,dF = \int_A y p\,dA = \int_A y \gamma h\,dA = \int_A y \gamma y \sin\theta\,dA$$

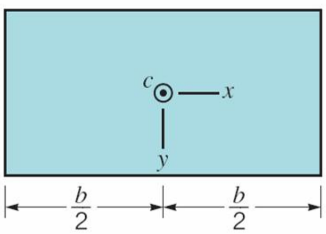
$$y_{CP} = \frac{\gamma \sin\theta \int_A y^2\,dA}{F_R} = \frac{\gamma \sin\theta I_x}{\gamma y_c A \sin\theta}$$

moment of inertia about x axis $I_x = \int_A y^2\,dA$

$$y_{CP} = \frac{I_x}{A y_c}$$

Relative and Absolute Values

- Handbook values give centroid of figure and moments of inertia about centroid



$$A = ba$$

$$I_{xc} = \frac{1}{12} b a^3$$

$$I_{yc} = \frac{1}{12} a b^3$$

$$I_{xyc} = 0$$

Figure 2.18(a), *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi

y_c for Rectangle

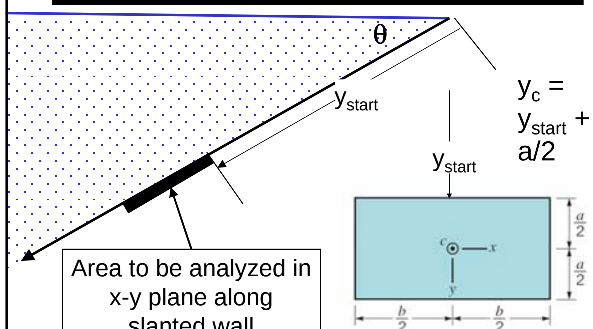


Figure 2.18(a), *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi

Moment of Inertia

- I_x for the moment of inertia is defined for starting at y = 0
  - For the orientation shown $y_c = y_{start} + a/2$
- For the rectangle as shown, $I_{xc} = ba^3/12$
- By parallel axis theorem: $I_x = I_{xc} + Ay_C^2$ (A = ab)
  - $y_c = y_{start} + a/2$

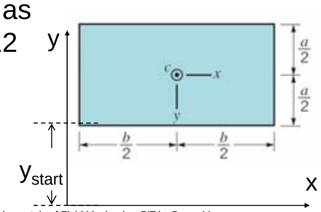


Figure 2.18(a), *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi

Moment of Inertia II

- Previous result for $y_{CP} = I_x/(Ay_c)$
- Parallel axis theorem: $I_x = I_{xc} + y_c^2A$
- Combined: $y_{CP} = I_{xc}/(Ay_c) + y_c$
- Note that $y_{CP} > y_C$
- For the rectangle as shown, $I_{xc} = ba^3/12$
- $y_c = y_{start} + a/2$
- $y_{CP} = ba^3/(12aby_c) + y_c = a^2/(12y_c) + y_c$

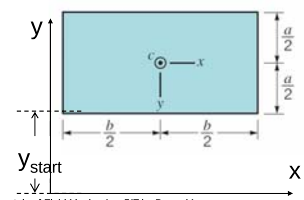


Figure 2.18(a), *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi

Circle Moment of Inertia

- Centroid at center of circle
- Remember to add y_{start} to get y_c
- **Problem:** A circular opening on a vertical wall in a water tank starts 5 m from the top and has R = 0.1 m. Find y_c and y_{CP} for the gate in this opening.
- **Solution:** $y_c = y_{start} + R$, $y_{CP} = I_{xc}/(Ay_c) + y_c$

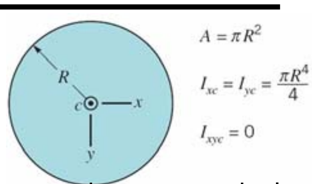


Figure 2.18(b), *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi

Circle Moment of Inertia II

- **Problem:** $y_{start} = 5$ m and R = 0.1 m.
- **Solution:** $y_c = y_{start} + R$ $y_{CP} = I_{xc}/(Ay_c) + y_c$
- $y_c = 5\text{ m} + 0.1\text{ m} = 5.1\text{ m}$

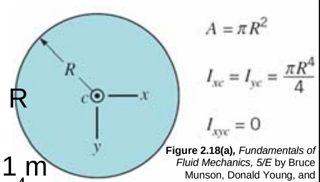


Figure 2.18(a), *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi

$$y_{CP} = \frac{I_{xc}}{Ay_c} + y_c = \frac{\frac{\pi R^4}{4}}{\pi R^2 y_c} + y_c = \frac{R^2}{4y_c} + y_c$$

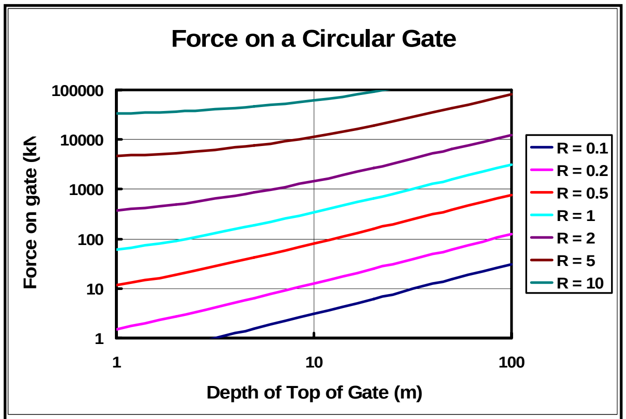
$$y_{CP} = \frac{(0.1\,m)^2}{4(5.1\,m)} + 5.1\,m = 5.1005\,m$$

Resultant Force

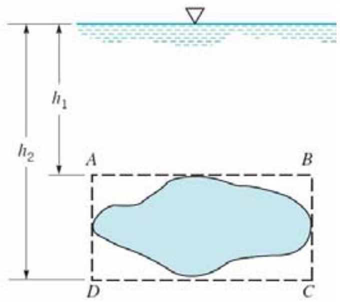
- Data and results for the circular gate: $y_{start} = 5.1$ m, R = 0.1 m, $y_c = 5.1$ m, vertical wall ($\theta = 90^o = \pi/4$)
- Find resultant force for water at 20°C ($\gamma = 9.789$ kN/m³)

$$F_R = \gamma y_c A \sin\theta = \frac{9.789\,kN}{m^3}(5.1\,m)\pi(0.1\,m)^2 \sin\left(\frac{\pi}{4}\right)$$

- $F_R = 1.11$ kN



Buoyancy



- A submerged object will weigh less because the pressure on its bottom is larger than the pressure on its top

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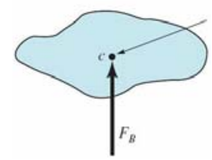
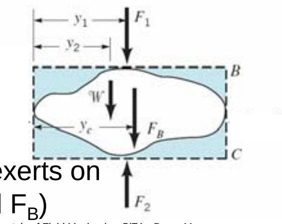
Buoyancy II



- Buoyant force, F_B, due to difference in pressure between top and bottom



- Analyze F_B by examining forces with solid not present
- Here F_B = force solid exerts on fluid (opposite of usual F_B)

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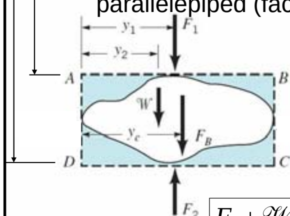
Buoyancy III

h_2 h_1

- Analyze a parallelepiped around the space the submerged object occupied
  - A = area of upper/lower surface of parallelepiped (facing top/bottom of slide)



- V_b = volume of body
- $V_f = A(h_2 - h_1) - V_b$ = volume of fluid
- $\mathcal{W} = \gamma V_f$ = weight of fluid

$$F_1 + \mathcal{W} + F_B = F_2$$

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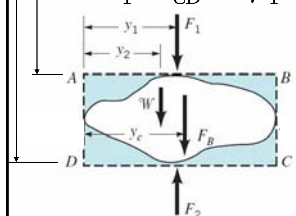
Buoyancy IV

h_2 h_1

$$F_1 + \mathcal{W} + F_B = F_2 \quad \Rightarrow \quad F_B = F_2 - F_1 - \mathcal{W}$$

$$\mathcal{W} = \gamma V_f = \gamma[A(h_2 - h_1) - V_b]$$

$$F_1 = P_{CD} A = \gamma h_1 A \qquad F_2 = P_{AB} A = \gamma A h_2$$



$$F_B = F_2 - F_1 - \mathcal{W} = \gamma A h_2 - \gamma A h_1 - \gamma[A(h_2 - h_1) - V_b]$$

- $F_B = \gamma V_b$

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Bouyancy V

- **Archimedes principle:** The buoyancy force, $F_{B,}$ on a submerged body is the specific weight of the fluid, γ, times the volume of the body, V_b, or $F_B = \gamma V_b$
- The buoyant force passes through the centroid of the submerged body
  - This result can be proved by analyzing moments of the forces used to determine F_B

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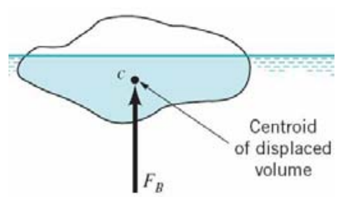
Buoyancy VI



- If the object is light enough, it will float on the top of the liquid
- This analysis ignores the weight of the top layer of fluid assumed to be air
  - Can consider buoyancy of an object be-tween two fluid layers with different densities

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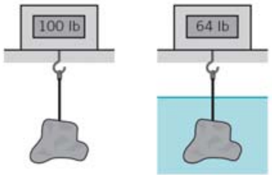
Problem



- An object weights 110 lb_f in air and 64 lb_f in water
- Find its volume and specific weight
- The difference in weights is the buoyant force, $F_B = 100\ lb_f - 64\ lb_f = 36\ lb_f = \gamma V_b$

$$V_b = \frac{F_b}{\gamma_w} = \frac{36\ lb_f}{\frac{62.4\ lb_f}{ft^3}} = 0.577\ ft^3 \qquad \gamma_b = \frac{W_{b,air}}{V_b} = \frac{100\ lb_f}{0.577\ ft^3} = \frac{173\ lb_f}{ft^3}$$

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25